Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topic: Inverse Trigonometric Function

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only are is correct. You have to select the correct answer which is the most appropriate.

1. If $x \in \left(\frac{3\pi}{2}, 2\pi\right)$, then the value of the expression $\sin^{-1}[\cos\{\cos^{-1}(\cos x)\} + \sin^{-1}(\sin x)]$, is

(a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{2}$

(c) 0 (d) π

2. The sum of the infinite terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{4}{33}\right) + \ldots$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

3. If $\sin^{-1}\sqrt{(x^2+2x+1)} + \sec^{-1}\sqrt{(x^2+2x+1)} = \frac{\pi}{2}$, $x \neq 0$, then the value of $2\sec^{-1}\left(\frac{x}{2}\right) + \sin^{-1}\left(\frac{x}{2}\right)$ is equal to

(a) $-\frac{3\pi}{2}$ (b) $\frac{3\pi}{2}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

4. The greatest of tan 1, $\tan^{-1} 1$, $\sin^{-1} 1$, sin 1, cos 1, is

(a) sin 1 (b) tan 1

(c) $\tan^{-1} 1$ (d) none of these

5. The value of '*a*' for which $ax^2 + \sin^{-1}(x^2-2x+2) + \cos^{-1}(x^2-2x+2) = 0$ has a real solution, is

(a) $-\frac{2}{\pi}$ (b) $\frac{2}{\pi}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

6. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ and $f(1) = 2$, $f(p+q) = f(p)\cdot f(q), \forall\ p, q \in R$, then $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{(x+y+z)}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$ is equal to

(a) 0 (b) 1

(c) 2 (d) 3

7. If the mapping $f(x) = ax + b, a > 0$ maps $[-1, 1]$ onto $[0, 2]$, then $\cot[\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18]$ is equal to

(a) $f(-1)$ (b) $f(0)$

(c) $f(1)$ (d) $f(2)$

8. The sum of the infinite series $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{6}}\right) + \sin^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}}\right) + \ldots + \ldots + \sin^{-1}\left(\frac{\sqrt{n}-\sqrt{(n-1)}}{\sqrt{\{n(n+1)\}}}\right) + \ldots$ is

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

9. If $[\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1}x] = 1$, where [.] denotes the greatest integer function, then x belongs to the interval

(a) [tan sin cos 1, tan sin cos sin 1]

(b) (tan sin cos 1, tan sin cos sin 1)

(c) $[-1, 1]$

(d) [sin cos tan 1, sin cos sin tan 1]

10. Solution set of $[\sin^{-1}x] > [\cos^{-1}x]$, where [.] denotes the greatest integer function, is

(a) $\left[\frac{1}{\sqrt{2}}, 1\right]$ (b) (cos 1, sin 1)

(c) [sin 1, 1] (d) none of these

11. If $[\cot^{-1} x] + [\cos^{-1} x] = 0$, where x is a non-negative real number and [.] denotes the greatest integer function, then complete set of values of x is
(a) $(\cos 1, 1]$ (b) $(\cot 1, 1)$
(c) $(\cos 1, \cot 1)$ (d) none of these

12. The number of solutions for the equation
$$2\sin^{-1}\sqrt{(x^2 - x + 1)} + \cos^{-1}\sqrt{(x^2 - x)} = \frac{3\pi}{2}$$ is
(a) 1 (b) 2
(c) 3 (d) infinite

13. The number of solutions of the equation
$\cos^{-1}(1 - x) + m\cos^{-1} x = \frac{n\pi}{2}$, where $m > 0$, $n \le 0$, is
(a) 0 (b) 1
(c) 2 (d) infinite

14. $\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}]$, is equal to
(a) $\sqrt{\left(\frac{x^2+1}{x^2+2}\right)}$ (b) $\sqrt{\left(\frac{x^2+2}{x^2+1}\right)}$
(c) $\sqrt{\left(\frac{x+2}{x^2+1}\right)}$ (d) $\sqrt{\left(\frac{x^2-1}{x^2+2}\right)}$

15. If $\cos^{-1}\left(\frac{x}{3}\right) + \cos^{-1}\left(\frac{y}{2}\right) = \frac{\theta}{2}$, then the value of $4x^2 - 12xy\cos\left(\frac{\theta}{2}\right) + 9y^2$ is equal to
(a) $18(1 + \cos\theta)$ (b) $18(1 - \cos\theta)$
(c) $36(1 + \cos\theta)$ (d) $36(1 - \cos\theta)$

16. $\cos^{-1}\left[\cos\left(-\frac{17}{15}\pi\right)\right]$ is equal to
(a) $-\frac{17\pi}{15}$ (b) $\frac{17\pi}{15}$
(c) $\frac{2\pi}{15}$ (d) $\frac{13\pi}{15}$

17. $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$ is equal to
(a) $\frac{5}{4}$ (b) $\frac{5}{16}$
(c) $-\frac{7}{17}$ (d) $\frac{7}{17}$

18. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to
(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) π

19. If $x + \frac{1}{x} = 2$, the principal value of $\sin^{-1} x$ is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) $\frac{3\pi}{2}$

20. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx$ is equal to
(a) -3 (b) 0
(c) 3 (d) -1

21. The value of $\sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)} + \cos^{-1}\left(\frac{\sqrt{12}}{4}\right) + \sec^{-1}(\sqrt{2})\right\}\right]$ is equal to
(a) 0 (b) $\pi/4$
(c) $\pi/6$ (d) $\pi/2$

22. The number of real solutions of
$$\tan^{-1}\sqrt{\{x(x+1)\}} + \sin^{-1}\sqrt{\{x^2 + x + 1\}} = \frac{\pi}{2}$$ is
(a) zero (b) one
(c) two (d) infinite

23. A solution of the equation
$\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$ is
(a) $x = 1$ (b) $x = -1$
(c) $x = 0$ (d) $x = \pi$

24. If x_1, x_2, x_3, x_4 are roots of the equation
$x^4 - x^3\sin 2\beta + x^2\cos 2\beta - x\cos\beta - \sin\beta = 0$, then $\sum_{i=1}^{4}\tan^{-1} x_i$ is equal to
(a) β (b) $\frac{\pi}{2} - \beta$
(c) $\pi - \beta$ (d) $-\beta$

25. If $\tan^{-1}\left\{\frac{\sqrt{(1+x^2)} - \sqrt{(1-x^2)}}{\sqrt{(1+x^2)} + \sqrt{(1-x^2)}}\right\} = \alpha$, then x^2 is equal to
(a) $\cos 2\alpha$ (b) $\sin 2\alpha$
(c) $\tan 2\alpha$ (d) $\cot 2\alpha$

26. If $x^2 + y^2 + z^2 = r^2$, then
$\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{zx}{yr}\right)$ is equal to
(a) π (b) $\frac{\pi}{2}$
(c) 0 (d) none of these

27. $-\frac{2\pi}{5}$ is the principal value of
(a) $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)$ (b) $\sin^{-1}\left(\sin\frac{7\pi}{5}\right)$
(c) $\sec^{-1}\left(\sec\frac{7\pi}{5}\right)$ (d) $\sin^{-1}\left\{\sin\left(\frac{2\pi}{5}\right)\right\}$

28. The value of
$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is equal to
(a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{12}$
(c) $\frac{3\pi}{4}$ (d) $\frac{13\pi}{12}$

29. If $\sum_{i=1}^{2n} \sin^{-1} x_i = n\pi$, then $\sum_{i=1}^{2n} x_i$ is equal to

(a) n (b) $2n$

(c) $\frac{n(n+1)}{2}$ (d) none of these

30. The inequality $\sin^{-1}(\sin 5) > x^2 - 4x$ holds, if

(a) $x = 2 - \sqrt{(9-2\pi)}$

(b) $x = 2 + \sqrt{(9-2\pi)}$

(c) $x \in (2 - \sqrt{(9-2\pi)}, 2 + \sqrt{(9-2\pi)})$

(d) $x > 2 + \sqrt{(9-2\pi)}$

31. The sum of the infinite series $\cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \ldots$ is equal to

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) none of these

32. $\sin\{\cot^{-1}(\tan \cos^{-1} x)\}$ is equal to

(a) x (b) $\sqrt{(1-x^2)}$

(c) $\frac{1}{x}$ (d) none of these

33. The value of $\tan^2(\sec^{-1}2) + \cot^2(\text{cosec}^{-1}3)$ is

(a) 13 (b) 15

(c) 11 (d) none of these

34. The equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for

(a) all real values of a (b) $a < 1$

(c) $-\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$ (d) $-1 < a < 1$

35. The number of real solutions of (x, y), where $|y| = \sin x$, $y = \cos^{-1}(\cos x)$, $-2\pi \le x \le 2\pi$, is

(a) 2 (b) 1

(c) 3 (d) 4

36. The number of positive integral solutions of $\tan^{-1} x + \cot^{-1} y = \tan^{-1} 3$ is

(a) one (b) two

(c) three (d) four

37. The value of $\cos^{-1}(\cos 12) - \sin^{-1}(\sin 12)$ is

(a) 0 (b) π

(c) $8\pi - 24$ (d) none of these

38. The smallest and the largest values of $\tan^{-1}\left(\frac{1-x}{1+x}\right)$, $0 \le x \le 1$ are

(a) $0, \pi$ (b) $0, \frac{\pi}{4}$

(c) $-\frac{\pi}{4}, \frac{\pi}{4}$ (d) $\frac{\pi}{4}, \frac{\pi}{2}$

39. If $-1 < x < 0$, then $\sin^{-1} x$ equals

(a) $\pi - \cos^{-1}\{\sqrt{1-x^2}\}$ (b) $\tan^{-1}\left\{\frac{x}{\sqrt{(1-x^2)}}\right\}$

(c) $-\cot^{-1}\left\{\frac{\sqrt{(1-x^2)}}{x}\right\}$ (d) $\text{cosec}^{-1} x$

40. The value of $\sin^{-1}(\sin 10)$ is

(a) 10 (b) $10 - 3\pi$

(c) $3\pi - 10$ (d) none of these

41. If a, b are positive quantities and, if

$$a_1 = \frac{a+b}{2},\ b_1 = \sqrt{a_1 b},$$

$$a_2 = \frac{a_1 + b_1}{2},\ b_2 = \sqrt{a_2 b_1} \text{ and so on, then}$$

(a) $a_\infty = \frac{\sqrt{(b^2-a^2)}}{\cos^{-1}\left(\frac{a}{b}\right)}$ (b) $b_\infty = \frac{\sqrt{(b^2-a^2)}}{\cos^{-1}\left(\frac{a}{b}\right)}$

(c) $b_\infty = \frac{\sqrt{(a^2-b^2)}}{\cos^{-1}\left(\frac{b}{a}\right)}$ (d) none of these

42. The value of $\tan^{-1}\left(\frac{c_1 x - y}{c_1 y + x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_2 c_1}\right) + \tan^{-1}\left(\frac{c_3 - c_2}{1 + c_3 c_2}\right) + \ldots + \tan^{-1}\left(\frac{1}{c_n}\right)$ is equal to

(a) $\tan^{-1}\left(\frac{y}{x}\right)$ (b) $\tan^{-1}\left(\frac{x}{y}\right)$

(c) $-\tan^{-1}\left(\frac{x}{y}\right)$ (d) none of these

43. The value of

$$\sin^{-1}\left\{(\sin \pi/3)\frac{x}{\sqrt{(x^2+k^2-kx)}}\right\} - \cos^{-1}\left\{\cos\frac{\pi}{6}\frac{x}{\sqrt{(x^2+k^2-kx)}}\right\}$$

$\left(\text{where } \frac{k}{2} < x < 2k, k > 0\right)$ is

(a) $\tan^{-1}\left(\frac{2x^2 + xk - k^2}{x^2 - 2xk + k^2}\right)$

(b) $\tan^{-1}\left(\frac{x^2 + 2xk - k^2}{x^2 - 2xk + k^2}\right)$

(c) $\tan^{-1}\left(\frac{x^2 + 2xk - 2k^2}{2x^2 - 2xk + 2k^2}\right)$

(d) none of the above

44. The value of $\tan\left\{\left(\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right)\right\}$ is

(a) $\frac{2}{3\sqrt{5}}$ (b) $\frac{2}{3}$

(c) $\frac{1}{\sqrt{5}}$ (d) $\frac{4}{\sqrt{5}}$

45. Sum infinite terms of the series

$$\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+\cot^{-1}\left(3^2+\frac{3}{4}\right)+\ldots \text{ is}$$

(a) $\frac{\pi}{4}$ (b) $\tan^{-1} 2$
(c) $\tan^{-1} 3$ (d) none of these

46. If $2\tan^{-1} x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is independent of x, then

(a) $x\in[1,\infty)$ (b) $x\in[-1,1]$
(c) $x\in(-\infty,-1]$ (d) none of these

47. If $\cot^{-1}\left(\frac{n}{\pi}\right)>\left(\frac{\pi}{6}\right)$, $n\in N$, then the maximum value of n is

(a) 1 (b) 5
(c) 9 (d) none of these

48. If $(\tan^{-1} x)^2+(\cot^{-1} x)^2=\frac{5\pi^2}{8}$, then x equals

(a) 0 (b) −1
(c) −2 (d) −3

49. The number of the positive integral solutions of

$$\tan^{-1} x+\cos^{-1}\left(\frac{y}{\sqrt{(1+y^2)}}\right)=\sin^{-1}\left(\frac{3}{\sqrt{10}}\right) \text{ is}$$

(a) 1 (b) 2
(c) 3 (d) 4

50. If $a_1, a_2, a_3, \ldots, a_n$ is an AP with common difference d, then

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

is equal to

(a) $\frac{(n-1)d}{a_1+a_n}$ (b) $\frac{(n-1)d}{1+a_1a_n}$
(c) $\frac{nd}{1+a_1a_n}$ (d) $\frac{a_n-a_1}{a_n+a_1}$

51. If $\tan\theta+\tan\left(\frac{\pi}{3}+\theta\right)+\tan\left(-\frac{\pi}{3}+\theta\right)=k\tan 3\theta$, then the value of k is

(a) 1 (b) $\frac{1}{3}$
(c) 3 (d) none of these

52. The principal value of

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right) \text{ is}$$

(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$

53. If $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$, then the two curves $y=\cos x$ and $y=\sin 3x$ intersect at

(a) $\left(\frac{\pi}{4},\frac{1}{\sqrt{2}}\right)$ and $\left(\frac{\pi}{8},\cos\frac{\pi}{8}\right)$
(b) $\left(-\frac{\pi}{4},\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{\pi}{8},\cos\frac{\pi}{8}\right)$
(c) $\left(\frac{\pi}{4},-\frac{1}{\sqrt{2}}\right)$ and $\left(\frac{\pi}{8},-\cos\frac{\pi}{8}\right)$
(d) $\left(-\frac{\pi}{4},\frac{1}{\sqrt{2}}\right)$

54. The solution of the inequality $(\cot^{-1} x)^2-5\cot^{-1} x+6>0$ is

(a) $(\cot 3, \cot 2)$ (b) $(-\infty, \cot 3)\cup(\cot 2, \infty)$
(c) $(\cot 2, \infty)$ (d) none of the above

Objective Questions Type II [One or more than one correct answers(s)]

In each of the questions below four choices of which are or more than one are correct. You have to select the correct answer(s) according.

1. The values of x satisfying $\sin^{-1} x+\sin^{-1}(1-x)=\cos^{-1} x$ are

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

2. If $\frac{1}{2}<|x|<1$, then which of the following are real?

(a) $\sin^{-1} x$ (b) $\tan^{-1} x$
(c) $\sec^{-1} x$ (d) $\cos^{-1} x$

3. $\sin^{-1} x>\cos^{-1} x$ holds for

(a) all values of x (b) $x\in\left(0,\frac{1}{\sqrt{2}}\right)$
(c) $x\in\left(\frac{1}{\sqrt{2}},1\right)$ (d) $x=0.75$

4. If $6\sin^{-1}(x^2-6x+8.5)=\pi$, then

(a) $x=1$ (b) $x=2$
(c) $x=3$ (d) $x=4$

5. Let $f(x)=e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)}$, then

(a) $f\left(\frac{8\pi}{9}\right)=e^{\frac{5\pi}{18}}$ (b) $f\left(\frac{8\pi}{9}\right)=e^{\frac{13\pi}{18}}$
(c) $f\left(-\frac{7\pi}{4}\right)=e^{\frac{\pi}{12}}$ (d) $f\left(-\frac{7\pi}{4}\right)=e^{\frac{11\pi}{12}}$

6. If $\alpha \le \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \le \beta$, then
(a) $\alpha = 0$ (b) $\beta = \pi/2$
(c) $\alpha = \pi/4$ (d) $\beta = \pi$

7. The greatest and least values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ are
(a) $\frac{\pi^3}{32}$ (b) $-\frac{\pi^3}{8}$
(c) $\frac{7\pi^3}{8}$ (d) $\frac{\pi}{2}$

8. The value of $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ is equal to
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) 0

9. The solution of the equation $\sin[2\cos^{-1}\{\cot(2\tan^{-1} x)\}] = 0$ are
(a) ± 1 (b) $1 \pm \sqrt{2}$
(c) $-1 \pm \sqrt{2}$ (d) none of these

10. α, β and γ are the angles given by $\alpha = 2\tan^{-1}(\sqrt{2} - 1)$, $\beta = 3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ and $\gamma = \cos^{-1}\left(\frac{1}{3}\right)$, then
(a) $\alpha > \beta$ (b) $\beta > \gamma$
(c) $\gamma > \alpha$ (d) none of these

11. Indicate the relation which is true
(a) $\tan|\tan^{-1} x| = |x|$ (b) $\cot|\cot^{-1} x| = |x|$
(c) $\tan^{-1}|\tan x| = |x|$ (d) $\sin|\sin^{-1} x| = |x|$

12. $\cos^{-1}\left(\sqrt{\frac{a-x}{a-b}}\right) = \sin^{-1}\left(\sqrt{\frac{x-b}{a-b}}\right)$ is possible, if
(a) $a > x > b$
(b) $a < x < b$
(c) $a = x = b$
(d) $a > b$ and x, takes any value

13. $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}\left\{\left(\frac{1}{3}\right)\tan\theta\right\}$, if
(a) $\tan\theta = -2$ (b) $\tan\theta = 0$
(c) $\tan\theta = 1$ (d) $\tan\theta = 2$

14. If the numerical value of $\tan\left\{\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right\}$ is $\frac{a}{b}$, then
(a) $a + b = 23$ (b) $a - b = 11$
(c) $3b = a + 1$ (d) $2a = 3b$

15. If $\operatorname{cosec}^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$, then x may be
(a) 1 (b) $-\frac{1}{2}$
(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

16. $2\cot^{-1} 7 + \cos^{-1}\left(\frac{3}{5}\right)$ is equal to
(a) $\cot^{-1}\left(\frac{44}{117}\right)$ (b) $\operatorname{cosec}^{-1}\left(\frac{125}{117}\right)$
(c) $\tan^{-1}\left(\frac{4}{117}\right)$ (d) $\cos^{-1}\left(\frac{44}{125}\right)$

17. If the equation $\sin^{-1}(x^2 + x + 1) + \cos^{-1}(\lambda x + 1) = \frac{\pi}{2}$ has exactly two solutions, then λ cannot have the integral value
(a) -1 (b) 0
(c) 1 (d) 2

18. The value(*s*) of x satisfying the equation $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ is/are given by (n is any integer)
(a) $n\pi - 1$ (b) $n\pi$
(c) $n\pi + 1$ (d) $2n\pi + 1$

19. If $\tan^{-1} y = 4\tan^{-1} x$, then y is infinite, if
(a) $x^2 = 3 + 2\sqrt{2}$ (b) $x^2 = 3 - 2\sqrt{2}$
(c) $x^4 = 6x^2 - 1$ (d) $x^4 = 6x^2 + 1$

20. If $\cos^{-1} x = \tan^{-1} x$, then
(a) $x^2 = (\sqrt{5} - 1)/2$
(b) $x^2 = (\sqrt{5} + 1)/2$
(c) $\sin(\cos^{-1} x) = (\sqrt{5} - 1)/2$
(d) $\tan(\cos^{-1} x) = (\sqrt{5} - 1)/2$

Linked-Comprehension Type

In these questions, passage (paragraph) has been given followed by questions based on each of the passages. You have to answer the questions based on the passage given.

PASSAGE 1

Let $f : A \to B$ be a function defined by $y = f(x)$ such that f is both one-one (Injective) and onto (surjective) (*ie*, bijective), then there exists a unique function $g : B \to A$ such that
$f(x) = y \Leftrightarrow g(y) = x, \forall x \in A$ and $y \in B$, then g is said to be inverse of f. Thus, $g = f^{-1} : B \to A = [\{f(x), x\} : \{x, f(x)\} \in f^{-1}]$.
If no branch of an inverse trigonometric function is mentioned, then it means the principal value branch of that function.

On the basis of above information, answer the following questions :

1. The value of $\cos(\tan^{-1}\tan 4)$ is

(a) $\frac{1}{\sqrt{17}}$ (b) $-\frac{1}{\sqrt{17}}$

(c) $\cos 4$ (d) $-\cos 4$

2. If x takes negative permissible value, then $\sin^{-1} x$ is equal to

(a) $\cos^{-1}\sqrt{(1-x^2)}$ (b) $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

(c) $\pi - \cos^{-1}\sqrt{(1-x^2)}$ (d) $-\pi + \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

3. If $\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$, then $\sin^{-1}(\sin x)$ is equal to

(a) x (b) $-x$

(c) $2\pi - x$ (d) $x - 2\pi$

4. If $x > 1$, then the value of $2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) $\frac{3\pi}{2}$

5. If $0 \le x \le 1$, then the least and greatest values of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ are

(a) $-\frac{\pi}{4}, \frac{\pi}{4}$ (b) $0, \frac{\pi}{4}$

(c) $\frac{\pi}{4}, \frac{\pi}{2}$ (d) $0, \pi$

PASSAGE 2

$$\sum_{r=1}^{n} \tan^{-1}\left(\frac{x_r - x_{r-1}}{1 + x_{r-1}x_r}\right) = \sum_{r=1}^{n} (\tan^{-1} x_r - \tan^{-1} x_{r-1}) = \tan^{-1} x_n - \tan^{-1} x_0, \forall\, n \in N$$

On the basis of above information, answer the following questions :

1. The sum to infinite terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \ldots + \tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right) + \ldots$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) none of these

2. The value of $\operatorname{cosec}^{-1}\sqrt{5} + \operatorname{cosec}^{-1}\sqrt{65} + \operatorname{cosec}^{-1}\sqrt{(325)} + \ldots$ to ∞ is

(a) π (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

3. The sum to infinite terms of the series $\cot^{-1}\left(2^2 + \frac{1}{2}\right) + \cot^{-1}\left(2^3 + \frac{1}{2^2}\right) + \cot^{-1}\left(2^4 + \frac{1}{2^3}\right) + \ldots$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\cot^{-1} 2$ (d) $-\cot^{-1} 2$

4. The sum to infinite terms of the series $\cot^{-1} 3 + \cot^{-1} 7 + \cot^{-1} 13 + \ldots$ is

(a) $\frac{\pi}{2}$ (b) $\cot^{-1} 2$

(c) $\tan^{-1} 2$ (d) none of these

5. The sum to infinite terms of the series $\tan^{-1}\left(\frac{2}{1-1^2+1^4}\right) + \tan^{-1}\left(\frac{4}{1-2^2+2^4}\right) + \tan^{-1}\left(\frac{6}{1-3^2+3^4}\right) + \ldots$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) none of these

PASSAGE 3

Principal values for inverse circular functions :

$x<0$	$x\geq 0$
$-\frac{\pi}{2}\leq \sin^{-1}x<0$	$0\leq \sin^{-1}x\leq \frac{\pi}{2}$
$\frac{\pi}{2}<\cos^{-1}x\leq \pi$	$0\leq \cos^{-1}x\leq \frac{\pi}{2}$
$-\frac{\pi}{2}<\tan^{-1}x<0$	$0\leq \tan^{-1}x<\frac{\pi}{2}$
$\frac{\pi}{2}<\cot^{-1}x<\pi$	$0<\cot^{-1}x\leq \frac{\pi}{2}$
$\frac{\pi}{2}<\sec^{-1}x\leq \pi$	$0\leq \sec^{-1}x<\frac{\pi}{2}$
$-\frac{\pi}{2}\leq \text{cosec}^{-1}x<0$	$0<\text{cosec}^{-1}x\leq \frac{\pi}{2}$

Ex. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$ not $\frac{2\pi}{3}$, $\tan^{-1}(-\sqrt{3})=-\frac{\pi}{3}$ not $\frac{2\pi}{3}$

On the basis of above information, answer the following questions :

1. The principal value of $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)+\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$ is

(a) $\frac{8\pi}{3}$ (b) $\frac{4\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

2. The principal value of $\sin^{-1}(\sin 5)-\cos^{-1}(\cos 5)$ is

(a) 0 (b) $2\pi-10$

(c) $-\pi$ (d) $3\pi-10$

3. The principal value of

$\tan^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right)+\cot^{-1}\cot\left(-\frac{3\pi}{4}\right)$ is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{-3\pi}{2}$ (d) 0

4. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos x)+\sin^{-1}(\sin x)\}]$, where $x\in\left(\frac{\pi}{2},\pi\right)$ is

(a) $\frac{\pi}{2}$ (b) $-\pi$

(c) π (d) $-\frac{\pi}{2}$

5. The number of solutions of the equation

$\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\sin^{-1}\left(\frac{2x}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$ is

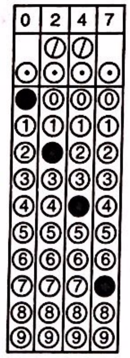
(a) 1 (b) 2

(c) 3 (d) infinite

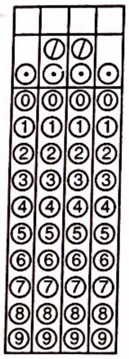
Numerical Grid-Based Problems

Solve the following problems and mark your response against their respective grids. Write your answer in the top row of the grid and darken the concerned numbers in the respective columns.

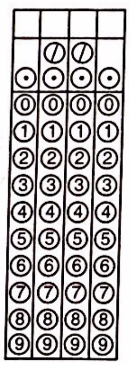
For example. If answer of a question is 0247, then



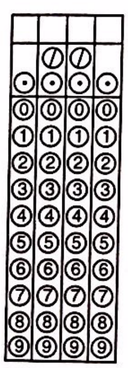
1. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, then the value of $2500(x+y+z)-\frac{216}{(x^3+y^3+z^3)}$ must be



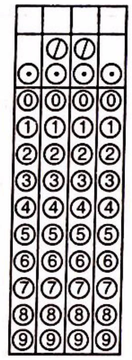
2. The value of $\frac{1}{\pi}\left\{216\sin^{-1}\left(\sin\frac{7\pi}{6}\right)+27\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+28\tan^{-1}\left(\tan\frac{5\pi}{4}\right)+200\cot^{-1}\left(\cot\left(\frac{-\pi}{4}\right)\right)\right\}$ must be



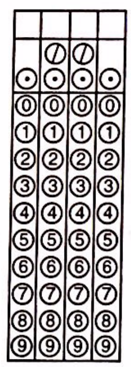
3. If $S=\sum_{r=1}^{50}\tan^{-1}\left(\frac{2r}{2+r^2+r^4}\right)$, then the value of $2550\cot S$ must be



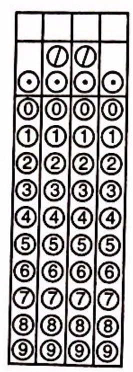
4. If $\tan^{-1}(x+1)+\tan^{-1}x+\tan^{-1}(x-1)=\tan^{-1}3$, then the value of (for $x<0$) $500x^4+270x^2+997$ must be



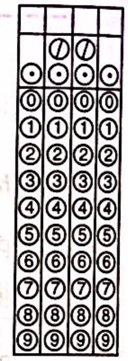
5. If $\lambda=\cos^4[\tan^{-1}\{\sin(\cot^{-1}5)\}]$, then the value of 3645λ must be



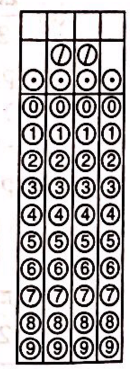
6. If $\theta=\cot^{-1}7+\cot^{-1}8+\cot^{-1}18$, then the value of $81\cot^4\theta$ must be



7. If $\lambda=\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$, then the value of $2890\lambda^2$ must be



8. If $\sin^{-1}x+\sin^{-1}y=\pi$ and, if $x=\lambda y$, then the value of $39^{2\lambda}+5^{\lambda}$ must be



Matrix-Match Type

Given below are Matching Type Questions, with two columns (each having some items) each . Each item of Column I has to be matched with the items of Column II, by encircling the correct match(es).

NOTE An item of Column I can be matched with more than one items of Column II. All the items of Column II have to be matched.

1. Observe the following columns :

	Column I		Column II
(A)	If principal values of $\sin^{-1}\left(-\frac{1}{2}\right)+\tan^{-1}(\sqrt{3})$ and $\cos^{-1}\left(-\frac{1}{2}\right)$ are λ and μ respectively, then	(P)	$\lambda+\mu=\frac{\pi}{2}$
		(Q)	$\mu-\lambda=\frac{\pi}{2}$
(B)	If Principal values of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ and $\cos^{-1}\left\{-\sin\left(\frac{5\pi}{6}\right)\right\}$ are λ and μ respectively, then	(R)	$\lambda+\mu=-\frac{\pi}{6}$
		(S)	$\mu-\lambda=\frac{5\pi}{6}$
(C)	If principal values of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ and $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$ are λ and μ respectively, then	(T)	$\lambda+\mu=\frac{5\pi}{6}$

(A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

2. Observe the following columns :

	Column I		Column II
(A)	If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, then	(P)	$x=y=z$
		(Q)	$xyz\geq 3\sqrt{3}$
(B)	If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$, then	(R)	$x+y+z=xyz$
(C)	If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$ and $x+y+z=\sqrt{3}$, then	(S)	$xyz\leq\frac{1}{3\sqrt{3}}$
		(T)	$xy+yz+zx=1$

(A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

3. Observe the following columns :

	Column I		Column II
(A)	If $2\tan^{-1}(2x+1)=\cos^{-1}(-x)$, then x is	(P)	$-\frac{1}{\sqrt{2}}$
		(Q)	0
(B)	If $2\cos^{-1}x=\sin^{-1}(2x\sqrt{(1-x^2)})$, then x is	(R)	$\frac{1}{\sqrt{2}}$
		(S)	$\frac{\sqrt{3}}{2}$
(C)	If $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$, then x is	(T)	1

Answers

Objective Questions Type I [Only one correct answer]

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (c)
11. (b)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (c)
21. (a)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (b)	**28.** (c)	**29.** (b)	**30.** (c)
31. (c)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (c)	**36.** (b)	**37.** (c)	**38.** (b)	**39.** (b)	**40.** (c)
41. (b)	**42.** (b)	**43.** (d)	**44.** (a)	**45.** (b)	**46.** (a)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (b)
51. (c)	**52.** (a)	**53.** (a)	**54.** (b)						

Objective Questions Type II [One or more than one correct answer(s)]

1. (a, b)	**2.** (a, b, d)	**3.** (c, d)	**4.** (b, d)	**5.** (b, c)
6. (a, d)	**7.** (a, c)	**8.** (c, d)	**9.** (a, b, c)	**10.** (b, c)
11. (a, b, d)	**12.** (a, b)	**13.** (a, b, c)	**14.** (a, b, c)	**15.** (a, c, d)
16. (a, b, d)	**17.** (a, c, d)	**18.** (a, b, c)	**19.** (a, b, c)	**20.** (a, c)

Linked-Comprehension Type

Passage 1 **1.** (d) **2.** (d) **3.** (d) **4.** (c) **5.** (c)

Passage 2 **1.** (a) **2.** (d) **3.** (c) **4.** (d) **5.** (b)

Passage 3 **1.** (d) **2.** (c) **3.** (a) **4.** (d) **5.** (b)

Numerical Grid-Based Problems

1.	7	4	2	8
2.	0	1	3	9
3.	2	5	5	2
4.	1	7	6	7
5.	3	3	8	0
6.	6	5	6	1
7.	0	4	9	0
8.	1	5	2	6

Matrix-Match Type

1. A → (Q, T); B → (P, S); C → (Q, R)
2. A → (Q, R); B → (S, T); C → (P, S, T)
3. A → (Q); B → (R, S, T); C → (P, R)